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Beyond the Numerator: Visualizing the Equal Partitioning Barrier in Elementary Fraction Learning

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Abstract

Understanding fractions remains a persistent challenge for elementary students, particularly when translating verbal mathematical problems into visual representations. This study aims to analyze the visual-conceptual difficulties of elementary school students in understanding fractions, specifically the principles of equal partitions and equivalence fraction, through visual representations. Using a thematic content analysis within a descriptive qualitative design, six fifth-grade students were selected based on their mathematical representation skills. Instruments included an illustrated diagnostic test (6 figures) and semi-structured interviews. The results reveal that the majority of students exhibited significant misconceptions rooted not in an inability to see differences, but in a "counting schema". They defined fractions solely by counting shaded parts versus total parts, ignoring the requirement of equal area. Specifically, students failed to apply the equal partition principle to disproportionate images, made systematic denominator errors due to excessive focus on the numerator, and showed rigidity in recognizing equivalent fractions (e.g., rejecting 2/6 the same as 1/3). These findings confirm that a discrete (counting) rather than continuous (measuring) reasoning schema acts as a significant conceptual barrier. Therefore, this study recommends non-prototypical and dynamic visual training that explicitly disrupts the counting habit and forces students to verify equal partitioning before labeling a fraction.

Keywords: counting schema; equal partitioning; fraction misconception; magnitude schema; visual representation

INTRODUCTION

The understanding of fraction is a cornerstone of mathematics education, which forms the basis for developing mathematical reasoning and more complex problem-solving skills. Fractions are not merely a representation of parts of a whole, but also reflect fundamental principles of equivalence, comparison, and proportional reasoning that students will encounter in their academic journey. However, many elementary students experience significant difficulties in understanding this concept, especially the principle of equal partition, which is central to understanding fractions (Čadež & Kolar, 2018).

Fractions are fundamentally defined as representation of a part of a whole, encapsulating the relationship between the numerator and denominator. This part-to-whole construction is pivotal in understanding fractions, as it provides the basis for students to understand the concepts of division and proportionality (Wilkins & Norton, 2018). The numerator signifies the number of equal parts being considered, while the denominator signifies the total number of equal parts that make up the whole. This relationship is not merely numerical, but also conceptual, which is essential for advanced mathematical reasoning. In learning practice, students often encounter challenges in applying this conceptual definition, particularly when they fail to recognize the significance of both components in determining equivalence and size (Singh et al., 2021). When interacting with fractions, the ability to visualize and understand the part-whole relationship is crucial (Čadež & Kolar, 2018; Rifandi, 2017). For example, students may mistakenly interpret fractions by focusing solely on the numerator, which leads to misconceptions about equivalence and size comparisons. This superficial interaction with the concept of fractions can hinder their overall mathematical skills. To overcome these difficulties, an approach is needed that bridges between abstract numerical concepts and tangible understanding, one of which is through visual representation.

Visual representations have proven to be effective tools in helping students understand fractions (Lerman, 2020). Models such as pie charts, bar models, and number lines facilitate students' to visualize the relationship between parts and wholes more clearly. Through this visual exploration, students can see firsthand how numerators and denominators work together to determine the size of a fraction. Thus, visual models are not only illustrative aids, but also a means of correcting common misconceptions, such as the tendency to only count the numerator without considering the denominator. In addition, visual representations play an important role in deepening the understanding of the concept of equivalence (Ertuna & Toluk Uçar, 2021; Hurst et al., 2020). For example, students can compare two different fractions and realize that even though the numerators are the same, the size of the fractions can be different because the denominators are not the same. This type of visualization reinforces the importance of the principle of equal division and strengthens proportional reasoning.

However, recent research has begun to question whether static area models (circle, squares, rectangles) may inadvertently reinforce the misconceptions they are meant to correct. For example, Wilkie & Roche (2023) that highlighted how teachers' preferences for particular fraction models (bars, circles, or sets) influence students' understanding. Unlike linear models (number lines) or dynamic digital representations, static area models allow students to count discrete 'parts' without considering the continuous magnitude. This creates a 'whole number bias' where students view fractions as a collection of countable units rather than as a proportional relationship (Di Lonardo Burr et al., 2022; Gabriel et al., 2023). Furthermore, the rise of virtual manipulatives and interactive fractions has shown the potential to disrupt this bias by allowing students to visibly resize parts and see equivalence dynamically (Xu et al., 2024). This study addresses a gap in the current literature, while numerous studies have documented equal partition errors, few have distinguished between students who cannot see unequal parts versus those who see but ignore them because their working definition of a fraction is purely counting-based. By introducing the cognitive distinction between "counting schema" (discrete, whole-number-based) and "magnitude schema" (continuous, proportional), this study offers a deeper explanatory framework for why static visual tests often fail to diagnose the root of fraction misconceptions.

However, misunderstandings still frequently arise. One of the most critical areas of misunderstanding is the 'Equal Partitions Principle,' which states that for fractions to be equivalent, the parts must be equal in size (Pedersen & Bjerre, 2021). This principle is the foundation for students to develop a coherent understanding of the equivalence and comparison of fractions. However, many students struggle with this concept, often due to a reliance on superficial strategies, such as simply counting the numerators without considering the denominators (Castro-Rodríguez et al., 2022). Recent studies have attempted to address aspects of fraction equivalence and visual understanding. Pedersen & Bjerre (2021) distinguish between *proportional equivalence* and *unit equivalence*, showing that students' interpretations of fraction equivalence often neglect the underlying conceptual foundations. Xu et al. (2024) conducted a holistic investigation of fraction learning by examining fraction skills, misconceptions, math anxiety, and self-confidence, but they did not specifically focus on students' interpretations of equal partitions. These studies emphasize the importance of visual models but leave a gap regarding how students themselves interpret and conceptualize equal partitions.

Research has shown that misunderstandings related to the Equal Partitions Principle can manifest in various ways (Čadež & Kolar, 2018). For example, students may mistakenly believe that fractions with the same numerator are always equal, regardless of the denominator (Castro-Rodríguez et al., 2022; González-Forte et al., 2023). This misunderstanding can lead to incorrect conclusions and hinder their ability to effectively solve problems involving fractions. A critical analysis of student responses, particularly in relation to visual representations, reveals how these misconceptions are rooted in their interpretation of the part-to-whole relationship. By focusing on the Equal Partitions Principle, educators can identify specific areas where students are struggling and develop targeted interventions to address these misunderstandings. A recent systematic scoping review by Sari et al. (2024) consolidated research on fractional learning obstacles and confirmed that equal partitioning and equivalence remain the two most frequently cited as conceptual barrier across multiple grade levels. Ultimately, addressing these misconceptions through visual-conceptual models and targeted instruction can lead to better mathematical outcomes and a deeper understanding of fractions among elementary school students.

The theoretical framework underpinning this study integrates three complementary perspectives: (1) Behr et al., (1983) rational number subconstruct theory, which posits that complete understanding of rational numbers requires integration of multiple subconstructs namely part-whole, ratio, measure, operator, and quotient; (2) Duval (2006) semiotic representation theory, which emphasizes that mathematical understanding depends on the ability to coordinate different types of representations (visual, symbolic, verbal); and (3) the whole number bias as a misconception which explains how students overgeneralize counting principles from natural numbers to fraction. These three theoretical lenses coverge to inform the analysis of students' responses in this study. Specifically the part-whole and ratio subconstructs to guide our interpretation of how students understand the relationship between numerators and denominators. Despite the recognized importance of fractions in elementary school, there exists a notable gap in current research specifically addressing the 'Equal Partitions Condition.' While numerous studies have explored various aspects of fractional understanding, the nuances of how students perceive and interpret equal partitions remain underexamined (Pedersen & Bjerre, 2021). In fact, this aspect is fundamental to building a correct understanding of fractions (Sari et al., 2024).

This study aims to bridge the existing gap in literature by analysis and mapping of visual-conceptual misconceptions related to the equivalence condition in fractions among elementary school students. By examining how students interpret visual representations of equal partitions, this study aims to illuminate the specific misconceptions that contribute to their misunderstanding of fractions. Through this analysis, we seek to provide valuable insights that can inform instructional practices and enhance educational strategies aimed at improving students' comprehension of fractional concepts.

METHOD

This study employs a descriptive qualitative research approach, utilizing thematic analysis to explore and analyze elementary school students' understanding of fractional concepts, particularly the principle of equal partitions. The qualitative nature of the research allows for an in-depth examination of students' thought processes and the identification of prevalent misconceptions (Creswell & Creswell, 2023). Thematic content analysis facilitates the systematic categorization of student responses, enabling a nuanced understanding of how students interpret visual representations of fractions.

The research subjects consisted of six elementary school students, all of whom were in fifth grade. This age group was particularly relevant because students at this level are typically introduced to more complex fraction concepts and are expected to develop a basic understanding of equality and division in fractions. The subjects were selected based on their level of mathematical representation ability, which was diagnosed from a fraction story problem-solving test. The classification criteria were adapted from the mathematical representation indicators outlined by the National Council of Teacher of Mathematics (NCTM) (2000), which describes three key forms of representation namely visual (icon/diagrams), symbolic (mathematical notation), and verbal (explanations). This criteria were further modified and contextualized for fraction visualization task based on Suryaningrum et al. (2020) research who operationalized mathematical representation ability to solve fraction story problem-solving test into observable indicator. This criterion is important to ensure that the research sample represents a broad spectrum of understanding, ranging from students with good to poor representation skills, so as to provide rich data on the profile of visual misconceptions. The criteria for grouping subjects are presented in Table 1.

Table 1. Classification Criteria of Students' Mathematical Representation Skills

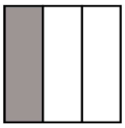
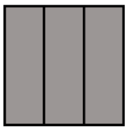
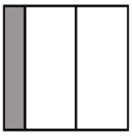
Category	Indicators
Good	• Students are able to create accurate and contextual images/visualizations (icons) according to the context of the question. • Students are able to create indices such as arrows, lines, or pointers to indicate the correct fractional relationships. • Students are able to write mathematical symbols or notation as supporting explanations in accordance with the images correctly.
Fair	• Students are able to create accurate and contextual images/visualizations (icons) according to the context of the question, but there are still shortcomings (incomplete or insufficient). • Students are able to create indices in the form of shaded fraction but without verbal explanation or not all relationships are explained. • Students use mathematical symbols but not entirely correct or complete.
Poor	• Students are unable to create accurate and contextual images/visualizations (icons) according to the context of the question. • Students did not use indexes or verbal connections between the components of the visualizations they created. • Students did not use mathematical symbols/notations.

Source: modification from (NCTM, 2000; Suryaningrum et al., 2020)

The criteria in Table 1 directly measure students' ability to translate symbols/problems into visual/iconic representations, which is the essence of conceptual understanding of fractions.

In addition, selecting subjects from different representation categories ensures that this study obtains rich and contrasting data on how students with different levels of representation interpret fraction visualizations. The research instrument used in this study was a test consisting of six different types of images designed to assess different aspects of students' understanding of fractions. Specifically, these images are described in Table 2.

Table 2. **The Research Instrument**

	Figure	Diagnostic Purpose	Cognitive Construct Targeted
Figure A		Benchmark for ideal equal partitions	Basic part-whole (control)
Figure B		Reveals denominator misconception/ (whole as $\frac{3}{3}$)	Whole number as fraction
Figure C		Main diagnostic for Equal Partition neglect	Continuous vs. discrete reasoning
Figure D		Equivalence recognition ($\frac{3}{6}$ vs $\frac{1}{2}$, is not $\frac{1}{3}$)	Equivalent fractions (non-target)
Figure E		Numerator focus ($\frac{1}{4}$ vs $\frac{1}{3}$)	Denominator salience
Figure F		Equivalent fractions ($\frac{2}{6} = \frac{1}{3}$)	Equivalence as proportional reasoning

The design of the six diagnostic figures was informed by multiple established sources. The equal partition diagnostic (Figure C) follows the task design principles from Zolfaghari (2023), who used circle, rectangle, and length models to assess children's fragmenting schemes, finding that students could partition into a given number of parts but not necessarily make those parts equal. The Figures A and E serve as positive and negative controls to verify basic part-whole understanding grounded in Behr et al.'s (1983) rational number subconstructs theory. The unequal partition distraction (Figure C) and the equivalence non-example (Figure D) are adapted from Čadež & Kolar (2018), who documented students' tendency to focus on numerators while ignoring denominators. The whole fraction task (Figure B) assesses unit fraction understanding adapted from Wilkins & Norton (2018).

The diagnostic test was administered individually in a quiet room. Each student was shown Figures A-F one at a time. For each figure, the student was asked: (1) "Does this picture show 1/3? Why or why not?" (2) "What fraction does this picture show?" (3) "How do you know if the parts are equal or not equal?" (probe added to assess awareness of equal partitioning). Semi-structured interviews were conducted immediately after each student completed the written responses, allowing the researcher to ask follow-up clarification questions such as: "You said this picture shows $\frac{1}{3}$. Can you show me which parts are equal?" or "You said this is not $\frac{1}{3}$. What would need to change for it to be $\frac{1}{3}$?" Interviews were audio-recorded and transcribed verbatim.

The data analysis technique involves an initial quantification of student responses, followed by systematic coding and grouping of rationales to map misconception profiles. Initially, responses will be categorized based on the correctness of the answers provided for each image type. This quantification will serve as a preliminary overview of the prevalence of misconceptions among the participants. Following this quantification, a thematic content analysis will be conducted to code student rationales. The coding process will involve identifying recurring themes in students' explanations, particularly focusing on their understanding (or misunderstanding) of the role of the numerator and denominator in determining equivalence. The systematic grouping of these codes will facilitate the development of misconception profiles, allowing for a comprehensive analysis of the different levels of understanding exhibited by the students.

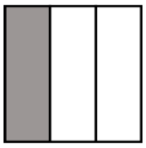
To ensure qualitative rigor, this study employed triangulation of data sources (written responses, interview transcript, researcher field notes). Member checking was conducted by returning the summary of each student's responses to the student for confirmation of intended meaning. Thick description of student rationales (presented verbatim in the Results section) allows readers to assess the transferability of findings to similar contexts. Through this methodologically sound approach, the study aims to illuminate the specific misconceptions that hinder students' comprehension of fractional concepts, particularly the principle of equal partitions, thereby informing future instructional strategies in mathematics education.

RESULT AND DISCUSSION

Result

The analysis of student responses revealed two distinct cognitive schemas that cut across individual items. Students with a "Counting Schema" (S2, S6, and partially S4) defined fractions exclusively as the ratio of shaded pieces to total pieces, regardless of whether those pieces were equal in area. For these students, any image with 1 shaded region out of 3 visible regions automatically represented $\frac{1}{3}$. Students with a "Magnitude Schema" (S1, S3, S5) demonstrated awareness that parts must be equal in size, but this schema was inconsistently applied – it was activated for equal partition items (Figure C) but often deactivated for equivalence items (Figure F). Only one student (S1) consistently applied a magnitude schema across all items, demonstrating integration of equal partitioning and equivalence principles. The following tables present individual responses, followed by a thematic synthesis organized by cognitive schema.

Table 3. Subject's Answers for Fraction Concept Test Item A

Figure	Subject	Answer	Fractions by subject	Subject's Rationale (Translated)
 A	**S1**	Yes	$\frac{1}{3}$	Because the image is divided into three parts, and only one is shaded.
	S2	Yes	$\frac{1}{3}$	Because there are 3 squares and 1 is shaded to become 1/3.
	S3	Yes	$\frac{1}{3}$	Because only one is shaded, the other two are not shaded.
	S4	Yes	$\frac{1}{3}$	Because only one is shaded, while there are three.
	S5	Yes	$\frac{1}{3}$	Because the image is divided into 3 and 1 is shaded.
	S6	Yes	$\frac{1}{3}$	Because only one is shaded to make up 1/3 and the rectangle is 3.

Responses to Image A (Ideal Representation) show an overall appropriate level of response, with all subjects stating that image A represents a fraction of $\frac{1}{3}$. All subjects explained with similar reasoning, namely that of the three parts, there was one shaded part representing $\frac{1}{3}$. However, the reasoning given by the subjects for this image still did not refer to the need for equal division, but rather focused on surface attributes such as the number of shaded parts out of the whole. Thus, even though the answers given are correct, the reasons given still cannot confirm that the subjects truly understand the principle of equal partitioning of fractions.

Table 4. Subject's Answers for Fraction Concept Test Item B

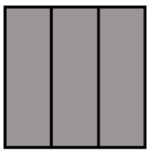
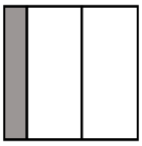
Figure	Subject	Answer	Fractions by subject	Subject's Rationale (Translated)
 B	**S1**	No	$\frac{3}{3}$	Because all of it is shaded, not just one.
	S2	No	$\frac{3}{3}$	Because all of it is shaded, it is 3/3 and not 1/3.
	S3	No	No Fraction	Because the box is completely shaded.
	S4	No	$\frac{3}{0}$	Because all of it is shaded.
	S5	No	$\frac{3}{3}$	Because all three are shaded.
	S6	No	$\frac{3}{1}$	Because all of it is shaded.

Table 4 shows the subjects' responses to a square image divided into three equal parts, all of which are shaded, mathematically representing $\frac{3}{3}$ or 1. Overall, the data shows that all subjects stated that image B was "not" a fraction of $\frac{1}{3}$, reasoning that the image showed that the shaded part was three of the three parts. However, even though the answers appear uniform in form, there is interesting variation in how subjects represent the fraction. The majority of subjects (S1, S2, S5) accurately represented the figure as $\frac{3}{3}$, giving the same reason: "Because it is all shaded." This shows that they understand that if the numerator (the shaded part) is equal to the denominator (the total number of parts), then the fraction is equal to one whole. Meanwhile, the other three subjects (S3, S4, and S6) showed difficulty with formal fraction notation, even though they had a correct verbal understanding. S3 stated that there was "no" fraction, reasoning that "the box is completely shaded." This is a correct verbal understanding (that it is a whole), but he was unable to represent the formal fraction symbol. When further interviews were conducted, there were differences in the interpretation of the fraction concept represented by image B among the six subjects. S1, S2, and S5 showed the correct interpretation by interpreting figure B as a fraction of $\frac{3}{3}$; whereas S3, S4, and S6 appeared to have made mistakes in interpreting the fractions from figure B as fractions of $\frac{3}{0}$, $\frac{3}{1}$, and no fraction.

Table 5. Subject's Answers for Fraction Concept Test Item C

Figure	Subject	Answer	Fractions by subject	Subject's Rationale (Translated)
 C	**S1**	No	Not $\frac{1}{3}$	Because there are fewer shaded parts than unshaded parts. Because the parts are not the same length and size.
	S2	Yes	$\frac{1}{3}$	Because there is 1 shaded part, it is $\frac{1}{3}$ of the box.
	S3	No	Not $\frac{1}{3}$	Because there is 1 box that is not the same, because the box is small.
	S4	No	Three per half	Because only half is shaded.
	S5	No	Three per half	Because the sizes are different.
	S6	Yes	$\frac{1}{3}$	Because there are three rectangles and only one is shaded, it is $\frac{1}{3}$.

Responses to figure C show varying levels of response among subjects. Overall, the data presented in Table 5 show significant misconceptions among subjects. Although the majority (S1, S3, S4, S5) had the correct intuition that the parts should be equal in size, two subjects (S2, S6) demonstrated a conceptual error by considering fractions only as the ratio of the number of shaded parts to the total number of parts, ignoring the fundamental requirement of fractions, namely equal parts.

Table 6. Subject's Answers for Fraction Concept Test Item D

Figure	Subject	Answer	Fractions by subject	Subject's Rationale (Translated)
 D	**S1**	No	$\frac{3}{6}$	Because there are three shaded parts, not two. If there were two, the fraction would be $\frac{1}{3}$.
	S2	No	$\frac{3}{6}$	Because there are 3 shaded parts and 6 squares, the fraction is $\frac{3}{6}$.
	S3	No	$\frac{3}{6}$	Because there are 3 shaded parts and 3 unshaded parts.
	S4	No	$\frac{3}{6}$	Because there are 3 shaded parts and 3 unshaded parts.
	S5	Yes	$\frac{1}{3}$	Because there are 3 shaded parts.
	S6	No	$\frac{3}{6}$	Because there are 6 blocks and 3 shaded parts.

The data from Table 6 shows the subjects' responses to a figure divided into six equal parts with three parts shaded, which mathematically represents $\frac{3}{6}$ or $\frac{1}{2}$. Overall, the data shows that the majority of subjects (5 out of 6) answered “No” when asked whether figure D represented $\frac{1}{3}$. Subjects S2, S3, S4, and S6 gave reasons for their answers based on the direct calculation of $\frac{3}{6}$ (three shaded parts out of a total of six parts), demonstrating mastery in determining the numerator and denominator. This contrasts with S1, who demonstrated a more specific conceptual understanding by explicitly stating that $\frac{1}{3}$ should be equal to $\frac{2}{6}$. On the other hand, S5 shows a significant misconception in understanding the relationship between visual representation and fractional values. Although visually the image clearly shows $\frac{3}{6}$, the subject stated that the image was $\frac{1}{3}$ simply because it was “shaded 3”. This highlights that S5 confused the fractional value $\frac{1}{3}$ with the numerator (3) without considering the proportions of the image.

Table 7. Subject’s Answers for Fraction Concept Test Item E

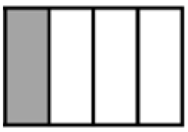
Figure	Subject	Answer	Fractions by subject	Subject’s Rationale (Translated)
 E	**S1**	No	$\frac{1}{4}$	Because there are 4 parts and only one is shaded.
	S2	No	$\frac{1}{4}$	Because there are 4 boxes, it does not correspond to $\frac{1}{3}$.
	S3	No	$\frac{1}{4}$	Because only one is shaded.
	S4	No	$\frac{1}{4}$	Because only one is shaded.
	S5	No	$\frac{1}{4}$	Because only one is shaded.
	S6	No	$\frac{1}{4}$	Because the rectangle is divided into 4 and only one is shaded.

Table 7 shows the subjects' responses to figure E, which is a rectangle divided into four equal parts, with one part shaded. Overall, the subjects' responses appear to be consistent and correct. All subjects answered “No” to the question of whether the image represented $\frac{1}{3}$ and correctly identified the fraction represented as $\frac{1}{4}$. The subjects' reasons for their answers also appear to be similar, with the rational core being that there are a total of four parts and only one is shaded. This shows that the subjects have a basic mastery of calculating fractions of a whole.

Table 8. Subject's Answers for Fraction Concept Test Item F

Figure	Subject	Answer	Fractions by subject	Subject's Rationale (Translated)
 F	**S1**	Yes	$\frac{2}{6} = \frac{1}{3}$	Because there are two shaded areas and there are 6 parts. So the fraction is $\frac{1}{3}$.
	S2	No	$\frac{2}{6}$	Because there are many squares and there are 2 shaded areas, the fraction is $\frac{2}{6}$.
	S3	No	$\frac{2}{6}$	Because there are 6 boxes and 2 shaded areas.
	S4	No	$\frac{2}{6}$	Because there are only two shaded areas.
	S5	No	$\frac{2}{6}$	Because only two are shaded, there should be 3 shaded.
	S6	No	$\frac{2}{6}$	Because the rectangle is divided into 6 parts and only 2 parts are shaded.

Table 8 shows the subjects' responses to figure F, which displays a rectangle divided into six equal parts, with two parts shaded. The subjects' responses to this image indicate a gap in their understanding of equivalent fractions. The majority of subjects (5 out of 6) answered "no," indicating that they did not recognize the equivalence of $\frac{2}{6} = \frac{1}{3}$. They only saw the ratio $\frac{2}{6}$ and concluded that it was not $\frac{1}{3}$. Their reasoning focused on simple calculations: "2 out of 6 parts are shaded, so it is $\frac{2}{6}$." S5 even showed an active misconception by stating that "it should be 3 shaded," even though the answer 1/3 only requires 2 parts to be shaded if there are 6 parts in total.

From the data, it is known that only S1 answered "Yes," and was the only one who demonstrated a correct understanding of equivalent fractions. The reason is explicitly stated: "because two are shaded and the part is 6. So this is a multiple of $\frac{1}{3}$". Thus, although all subjects successfully calculated the ratio $\frac{2}{6}$ correctly, only S1 was able to take the additional step of reasoning to identify that $\frac{2}{6}$ is a multiple, or equivalent, to $\frac{1}{3}$. The majority of the other subjects failed to identify this and focused on $\frac{2}{6}$, which led them to reject that figure F was equal to $\frac{1}{3}$ simply because the denominator and numerator were not equal to 1 and 3.

Cognitive Schema Typology

Students S2 and S6 consistently ignored equal partition requirements. In Figure C (unequal parts), both answered "Yes" because "there is 1 shaded part out of 3 boxes." When probed during interviews about whether the boxes were the same size, S2 replied: "It doesn't matter because it's still one box. A fraction is about how many boxes are shaded." This response confirms that their definition of a fraction was discrete and count-based, treating the image as a collection of countable units rather than a continuous area to be measured proportionally.

Next, inconsistent magnitude schema that appeared on S3, S4, S5. These students correctly rejected Figure C because "the parts are not the same size" (S3, S5) or "only half is shaded" (S4), demonstrating awareness of equal partitioning. However, they failed on Figure F (equivalence), rejecting 2/6 as 1/3. S5's rationale reveals the inconsistency: "It should be 3 shaded" – indicating that he was trying to match the numerator (1 in 1/3) rather than the proportional value. These students could detect unequal partitions but could not flexibly scale fractions to recognize equivalence. Their magnitude schema was triggered only when size differences were visually obvious (Figure C), not when proportional reasoning was required (Figure F).

Only S1 demonstrated consistent application of both equal partition and equivalence principles. S1 rejected Figure C ("the parts are not the same length and size") and accepted Figure F ("2/6 is a multiple of 1/3"). Interview probe: *"How did you know 2/6 is the same as 1/3?"* S1 responded: "Because if you cut each third into two pieces, you get six pieces and two are shaded." This response demonstrates the ability to mentally transform the unit (a key indicator of flexible proportional reasoning).

Discussion

A comprehensive analysis of items A to F reveals contradictions in the subjects' understanding of fractions. On the one hand, the subjects demonstrate a high level of mastery of the basic arithmetic aspects of fractions, namely the ability to calculate the visual ratio of the numerator and denominator. On the other hand, the findings highlight critical conceptual misconceptions related to two fundamental conditions in the definition of fractions: the condition of equal partitioning and the condition of equivalence.

The findings of this study extend beyond confirming known misconceptions (Kumalasari & Rahayuningsih, 2025) by introducing a cognitive distinction between discrete (counting) and continuous (measuring) reasoning about fractions. Students in the "Pure Counting Schema" typology (S2, S6) treated the visual fraction tasks as if they were counting discrete objects – like counting apples in a basket. This is consistent with the whole number bias documented by Di Lonardo Burr et al. (2022), where students overgeneralize counting principles from natural numbers to fractions. However, this study provides a novel contribution: even when students could verbally state that "parts must be equal" (as S3 and S5 did), they did not spontaneously apply this principle to equivalence tasks. This reveals that equal partitioning knowledge is not automatically transferred – it remains "inert knowledge" unless the task explicitly cues size comparison.

The data shows a significant gap between mechanical skills in calculating ratios and a deep conceptual understanding of fractions. In Item A ($\frac{1}{3}$) and Item E ($\frac{1}{4}$), all subjects correctly identified the fractions because these questions only required simple calculations: x shaded parts out of y total parts. The consistency in identifying $\frac{1}{3}$ in Item A and $\frac{1}{4}$ in Item E confirms that the subjects mastered the basic visual representation of unit fractions. However, this success did not apply when more complex concepts were tested. This was evident in the failure of the majority of subjects on Item C (condition of equal parts) and Item F (condition of equal values), which showed that their understanding was often limited to visual numerical interpretation and failed to cover the fundamental principles of fractions. These findings are in line with research by Kumalasari & Rahayuningsih (2025), which found that students lack a deep understanding of fractions, as seen when they are faced with questions that require more than just visual interpretation, such as fraction questions involving equal parts or equal values.

According to Behr et al. (1983), a complete understanding of rational numbers does not only involve one form of interpretation, but rather the integration of several subconstructs such as part-whole, ratio, measure, and quotient. The findings in this study also support the view that a complete understanding of rational numbers requires not only an understanding of each subconstruct of rational numbers, but also how they relate to each other in mathematical reasoning (Elias et al., 2020; Pittalis, 2025). As the data shows, students' mastery of the ratio and part-whole subconstructs (for example, the ability to refer to the fraction $\frac{1}{3}$ as “one of three parts” in item A) does not automatically indicate their mastery of the actual concept of fractions, because their understanding is still separate from other concepts of fractions, such as the concepts of equal partition and equivalent fractions.

The most fundamental misconception appears in Item C, which tests the equivalent condition (equal size) of the parts. Although the majority of subjects (S1, S3, S5) were able to reject the image as $\frac{1}{3}$ for the correct reason (‘the sizes are different’ or ‘they are not the same size’), this response reveals a misconception among the other subjects. Subjects S2 and S6 incorrectly answered ‘Yes’ because they only counted the number of partitions (one shaded out of three parts) without considering that the sizes of the parts were different. For these subjects, fractions were only seen as a process of counting the number of boxes, ignoring the key definition that fractions only apply if the whole is divided into equal parts (equipartition). This failure shows that their visual understanding of ‘boxes’ as valid parts was more dominant than their conceptual understanding of ‘equal size’ (Powell et al., 2022).

This finding also confirms the views of Behr et al. (1983) and Čadež & Kolar (2018) that the interpretation of parts-to-whole in rational numbers depends directly on the ability to divide into equal parts. This means that students' errors in recognizing that the parts in the image are not equal in size, as in item C, indicate a failure to meet the basic requirements of a valid fraction. When subjects only count the number of parts without taking into account the equal size of the parts, it indicates that the concept of equal partitioning has not been well internalized. This misunderstanding leads to invalid fraction representations, because a true fraction requires the whole to be divided into parts with equal area or size, not just any number of parts (Čadež & Kolar, 2018).

In addition, the concept of equivalence is also a weak point for students in understanding fractions. Data on Item F ($\frac{2}{6}$ vs $\frac{1}{3}$) reveals the subjects' vulnerability in understanding fraction equivalence. Although all subjects were able to identify the visual ratio as $\frac{2}{6}$, five of the six subjects rejected the image as $\frac{1}{3}$ and answered ‘No’. This rejection indicates that they are treated $\frac{2}{6}$ as a nominal number different from $\frac{1}{3}$, rather than as an equivalent quantity. Research shows that many students fail to understand the concept of equivalent fractions, viewing fractions as tied to their visible numerator and denominator (2 and 6), thereby hindering their ability to mentally simplify or connect fractions such as $\frac{2}{6}$ and $\frac{1}{3}$ to verify their equivalent values (Girgin & Altay, 2023). Only the S1 subject demonstrated a correct understanding, rationalizing that $\frac{2}{6}$ is a ‘multiple of $\frac{1}{3}$’, proving that he had shifted from thinking of fractions as ‘parts of a whole’ to thinking of fractions as equivalent numerical values. This reaction demonstrates a tendency to treat fractions as numbers with different values (the number $\frac{2}{6}$ is different from $\frac{1}{3}$), hindering their ability to reason about simplification. This misunderstanding is widespread, with studies showing that students often reject visual representations of equivalent fractions if the numbers are not exactly the same, demonstrating a lack of understanding that different fractions can represent the same value (Girgin & Altay, 2023).

This difference shows that although subjects have mastered procedural rational representation, they have not yet mastered the relationships between the subconcepts underlying fractions. Behr et al. (1983) explain that the ability to see equivalence and equal partitioning are two complementary cognitive mechanisms that form the basis for the formation of more complex fraction concepts. Therefore, failure to understand one of them will have a direct impact on misconceptions about the whole concept of fractions. Furthermore, the tendency of students to rely on surface visual attributes, such as counting the number of shaded boxes without considering size similarities, can be explained by the concept of visual-perceptual distracters (Picon et al., 2019). Behr et al. (1983) note that various types of perceptual cues can negatively affect children's thinking patterns and interfere with their logical thinking processes. This condition is evident when subjects reject or accept images based on misleading visual perceptions rather than logical mathematical reasoning. Thus, the failure of some subjects on items C and F is not only due to a lack of formal knowledge, but also because of the dominance of visual perception over conceptual thinking.

Although most subjects succeeded in basic questions, the data showed inconsistencies and misconceptions related to formal fraction notation and the concept of the whole. In Item B (fully shaded image), most subjects could verbally understand that the shaded part was the whole, but two subjects still showed difficulty in presenting the correct notation. Subject S4 wrote $\frac{3}{0}$ and Subject S6 wrote $\frac{3}{1}$. These errors indicate that the students did not understand that the denominator indicates the number of equal parts of a fraction. In addition, Subject S5 on Item D, who answered 'Yes' for the $\frac{3}{6}$ fraction image, showed that the subject was still confused about distinguishing between numbers that indicate the numerator and denominator representing different parts. Confusion between the numerator and denominator, as seen when students mislabel the parts or fail to distinguish their roles, is a common problem in early fraction learning (Di Lonardo Burr et al., 2022).

Therefore, as emphasized by Behr et al. (1983), the division and subconstruction of parts-to-wholes is the basis for studying other subconstructions of rational numbers, and the ratio subconstruction is the most natural way to introduce the concept of equivalence. Therefore, the pedagogical focus in fraction learning should be directed at strengthening two fundamental things, namely understanding equal partitioning and understanding the equivalence of fractions. Failure in these two aspects not only causes local conceptual errors, but also hinders students' cognitive development towards a more abstract and flexible understanding of rational numbers (F. Y. Sari et al., 2022).

This study successfully revealed the gap between students' procedural skills in calculating fraction ratios and their conceptual understanding of basic concepts such as equal partitioning and equivalence. The main strength of this study is its ability to identify fundamental misconceptions that hinder students' understanding, which can be used as a basis for designing more effective fraction teaching. However, the limitation of this study is a limitation of the study's sample size (N=6) for statistical generalization, but it is appropriate for the qualitative aim of identifying cognitive typologies (Creswell & Creswell, 2023). The thick description of each student's reasoning allows for analytical generalization, the findings (counting schema vs. magnitude schema) can be transferred to other elementary contexts where similar static visual tests are used. Future research should test whether the three typologies identified here (pure counting, inconsistent magnitude, integrated magnitude) replicate in larger samples and whether they predict performance on number line tasks. In addition, this study has not fully explored external factors such as teaching methods that may influence students' understanding.

CONCLUSION

Conclusion

In conclusion, this study has illuminated that the critical conceptual barrier in elementary students' understanding of fractions is not simply a lack of knowledge about equal partitions, but rather the dominance of a discrete "counting schema" over a continuous "magnitude schema." Students who failed on Figure C did so not because they could not see size differences but because their working definition of a fraction was exclusively "shaded parts out of total parts," a definition that treats fractions as whole numbers. This disconnect between visual perception and mathematical definition limits students' ability to recognize equivalence and compare fractional magnitudes.

Recommendations

To address this misconception, it is recommended that teachers use visual aids, such as circle diagrams and bar models, to teach the difference between equal and unequal divisions so that students can more clearly understand the importance of both parts in fractions. In addition, teachers also need to design activities that encourage students to compare fractions with equal and unequal parts while discussing their reasons for the size of the parts. Encouraging reflection through peer discussions and self-assessment is also important to help students reflect on their thinking processes and gain a deeper understanding of the material. Further research needs to focus on developing more sophisticated diagnostic tools to assess students' visual-conceptual understanding of fractions, as well as evaluating the effectiveness of interventions that integrate visual-conceptual models, in order to improve students' understanding of the concept of fractions.

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