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Number Sense and Mental Arithmetic in Terms of Mathematical Ability: A Case Study on Elementary School Students

Ade Firma Handriyani Putri
Neni Mariana*
Rooselyna Ekawati
[1,2]Pendidikan Dasar, Fakultas Ilmu Pendidikan, Universitas Negeri Surabaya
[3]Pendidikan Matematika, Fakultas Matematika dan IPA, Universitas Negeri Surabaya –
Jl. Lidah Wetan Kec. Lakar Santri Surabaya, 60213
*Corresponding author, E-mail: nenimariana@unesa.ac.id

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Abstract
This descriptive qualitative case study examines the number-sense and mental-arithmetic profiles of six fifth-grade students across varying mathematical ability levels. Data were collected through written tests, semi-structured interviews, and work documentation, then analyzed using data reduction, display, and verification. Findings revealed that high-ability students demonstrated mastery through flexibility, accuracy, and efficiency. Moderate-ability students met most criteria but showed inconsistencies with complex tasks, while low-ability students faced substantial difficulties and displayed unstable strategies. These results emphasize the necessity of differentiated instruction. To foster foundational numerical competencies, teachers should implement regular mental math activities, such as daily number discussions, to practice flexible estimation. Furthermore, providing scaffolded support—such as gradually reducing the use of scratch paper—is essential for low-ability students to alleviate mental strain and strengthen memory for mental calculations. This structured approach ensures effective development of mathematical proficiency across all student profiles. This approach ensures foundational numerical competencies are developed effectively across all student profiles.

Keywords: elementary school; mathematical abilities; mental arithmetic; number sense

INTRODUCTION

Mathematics is an essential aspect of human life and cannot be separated from daily activities. It is widely believed that mathematics fosters logical, analytical, critical, and systematic thinking skills (Putri et al., 2024). Numbers and arithmetic operations constitute fundamental mathematical content in elementary school, playing a crucial role in developing students' computational abilities for everyday tasks (Farida et al., 2014). An understanding of numerical concepts can be strengthened by cultivating sensitivity to numbers, commonly referred to as number sense.

Number sense is defined as an individual's ability to understand the meaning of numbers, recognize relationships among numerical units, compare magnitudes, anticipate the effects of various mathematical operations, and make use of appropriate reference points (Mcintosh et al., 1992; R. Reys et al., 1999; Yang, 2019). It involves the use of efficient strategies—such as mental computation and estimation—to address numerical tasks. For this reason, number sense is regarded as a crucial component of mathematical learning (Er & Artut, 2025), enabling students to apply their numerical knowledge across a range of contexts, including mathematical problem-solving (Kangjeng et al., 2024).

Numbers, which historically served as tools for calculation, were once represented using devices such as “SEMPOA”. In the twentieth century, research on brain development led to theories that shifted numerical representation from the physical *sempoa* to mental imagery, allowing calculations to be performed more quickly while simultaneously supporting cognitive development (Ismarti, 2012). In this context, the brain becomes the central factor in the success of numerical computation, as the calculation process relies entirely on cognitive activity (Fahrurrozi et al., 2021).

Literally, the term mental arithmetic consists of the word “mental,” referring to the mind, and “arithmetic,” referring to calculation. Conceptually, mental arithmetic can therefore be understood as a process of performing calculations by relying solely on the brain, without the aid of external tools such as calculators, abacuses, computers, or even pencils (Feronika et al., 2025). The importance of mental arithmetic lies in its role in supporting long-term mastery of mathematics across higher levels of education. The ability to compute using mental strategies can foster students’ capacity to apply numerical concepts in everyday contexts (Fahrudin & Fathani, 2025).

Number sense and mental arithmetic are two essential supporting components in mathematical learning, serving as prerequisites for mastering numerically oriented fields such as science, technology, and economics. This claim is reinforced by previous studies that highlight the significant contributions of number sense and mental arithmetic to students’ mathematical proficiency. Gökçe (2023), for example, found that number-sense skills are key indicators that contribute to students’ mathematical achievement. Other studies have likewise reported that higher number-sense abilities positively influence and enhance problem-solving performance (Ramdani et al., 2023), improve overall mathematical ability (Maghfirah, 2019), and affect mathematical learning outcomes (Mucti & Nurmala, 2020; Yuniarti, 2023). Research on mental-arithmetic skills also demonstrates its critical contribution. Studies by Feronika (2025) and Fahrurrozi (2021) emphasize that strengthened mental-arithmetic abilities can significantly reduce mathematical anxiety and phobia while simultaneously increasing students’ learning motivation.

Even though number sense and mental arithmetic are clearly important, a gap remains in current research. Most previous studies have only looked at these skills separately or focused mostly on how they affect general math scores. There is a lack of detailed qualitative research that explores both skills together to see how students actually use them at different math ability levels. Specifically, in elementary education, we still do not know enough about the specific thinking strategies, struggles, and mistakes made by high-, moderate-, and low-ability students. Filling this gap is important because understanding how these students think is necessary to help teachers create better, differentiated teaching methods.

With these considerations in mind, this study sets out to explore the topic entitled **“Number Sense and Mental Arithmetic in Terms of Mathematical Ability: A Case Study on Elementary School Students.”** This research integrates two key competencies—number sense and mental arithmetic—specifically, we are looking at how these skills manifest in elementary school students across different levels of mathematical ability. Furthermore, the study explores in depth each ability level along with the corresponding profiles of number-sense and mental-arithmetic skills.

METHODS

Research Design

This study seeks to explore the number sense and mental arithmetic skills of fifth-grade elementary school students in the topic of numbers and computation. Using a qualitative approach, it produces a descriptive analysis of students' number sense and mental arithmetic profiles across three levels of mathematical ability. These abilities are examined in a natural and comprehensive manner and presented in a narrative description (Cohen et al., 2018). The focus of the research lies in a comparative analysis of students with high, moderate, and low mathematical abilities, integrating both number sense competence and mental arithmetic performance.

Data Collection

Data collection was conducted at SD Muhammadiyah 1 Gresik with fifth-grade students during the first semester of the 2025/2026 academic year. The research subjects consisted of six students representing each category of mathematical ability. Data was gathered in October 2025. The selection of participants was based on the following considerations: (1) representation of each mathematical ability category, with two students assigned to each group; (2) sufficient communication skills to ensure that the exploration of number sense and mental arithmetic could proceed effectively; and (3) fifth-grade students were chosen because they have had a relatively long period of learning experience in mathematics. Furthermore, this intentionally small sample size was chosen to facilitate an in-depth, rich, and detailed qualitative exploration of each individual's cognitive processes, which is a fundamental characteristic of case study research.

Research Instrument

The data collection instruments used in this study consisted of several components: (1) a mathematical proficiency test to classify the research subjects, (2) a diagnostic test assessing number sense and mental arithmetic skills, (3) an interview guide, and (4) an observation sheet. To ensure the credibility and reliability of the data, all measurement tools underwent a rigorous expert validation process. The expert panel consisted of two university mathematics lecturers and one fifth-grade mathematics teacher. Following their comprehensive evaluation of the content, construct, and language clarity, the validators formally declared that the finalized instruments are valid, methodologically robust, and entirely suitable for measuring the students' number sense and mental arithmetic capabilities in this research. As the primary instrument in the data collection process, the number sense diagnostic test that was developed encompasses all components of number sense adapted from McIntosh et al. (1992), McIntosh et al. (1997), and Singh (2009), which will be presented in Table 1.

Table 1. Components of Number Sense

No.	Component	Description	Items
1.	Understanding number concepts	Involves comprehension of fundamental number meanings, including place-value systems, number patterns, and positional value (Yang & Li, 2008).	1
2.	Using multiple representations	Identifying numerical equivalence through various forms of representation, such as fractions, decimals, percentages, and number lines (Yang & Sianturi, 2020).	2
3.	Understanding the effects of operations	Comprehending the impact and meaning of mathematical operations, particularly how these operations influence numbers (Yang & Lin, 2015).	3
4.	Using equivalent representations	Applying arithmetic properties to transform an expression into an equivalent form that is easier to compute.	4
5.	Using calculation strategies	Employing mental strategies to compute and perform addition without relying on written methods (R. Reys et al., 1999).	5

In addition to the number sense components, this study also examines the elements of mental arithmetic skills, which constitute an essential part of the main data collection instrument. Mental arithmetic skills are understood as the ability to perform calculations rapidly and accurately without the use of writing tools or calculating devices. These skills provide insight into students' fluency, efficiency, and the cognitive strategies they employ when processing mathematical operations mentally.

Table 2. Components of Mental Arithmetic

No.	Component	Description	Items
1.	Accuracy of results	Refers to the ability to produce correct answers in mental calculations. This component evaluates the extent to which students can carry out mathematical operations without errors despite not using any calculating tools.	6
2.	Use of mental strategies	Describes students' ability to select and apply efficient cognitive strategies—such as number decomposition, rounding, or using number facts—to solve calculations mentally.	7
3.	Estimation ability	Refers to the skill of reasonably approximating the results of calculations to values close to the actual answer. Estimation enables students to make quick decisions in situations that do not require exact results.	8
4.	Ability to solve multi-step problems	Indicates the capacity to process and solve mathematical problems that require several sequential calculation steps while maintaining a logical flow of mental reasoning.	9
5.	Consistency in calculation	Refers to the ability to sustain stable calculation performance, both in terms of speed and accuracy, across various types of operations and conditions.	10

Data Analysis

After all data were collected, the analysis was carried out through three systematic stages proposed by Miles, Huberman, and Saldaña (2014): data reduction, data display, and conclusion drawing. In the data reduction stage, the researchers first transcribed the interview recordings and sorted the raw data. During this step, an open coding process was applied, where specific codes were assigned to the students' answers and interview transcripts. These codes were then grouped into specific categories based on the indicators of number sense and mental arithmetic. Next, in the data display stage, the categorized data were organized into clear narratives to present the skill profiles of high-, moderate-, and low-ability students. Finally, during the conclusion drawing and verification stage, the researchers interpreted these patterns to answer the research questions, constantly checking the conclusions against the raw data to ensure accuracy.

To ensure the quality and trustworthiness of the study, the researchers applied the criteria established by Lincoln and Guba (1985), focusing on credibility, dependability, and confirmability. Credibility was achieved through methodological triangulation (Creswell & Poth, 2018), where the researchers carefully compared data from the written test results, interview transcripts, and students' scratch papers to ensure the findings were consistent. Dependability was maintained by keeping a clear audit trail, meaning every step of the research process, from data collection to analysis, was well-documented so it could be evaluated by others. Lastly, to establish confirmability and prevent researcher bias, every conclusion was strictly based on the actual data. Direct quotes from the students and examples of their written work were actively used as clear evidence to support the final interpretations.

RESULT AND DISCUSSION

Primary data collection was carried out in October of the 2025/2026 academic year at SD Muhammadiyah 1 Gresik, involving fifth-grade students as research subjects. The study began with the administration of an initial mathematics test used as the basis for selecting participants. Based on the test results, six students were identified to represent three categories of mathematical ability: two students with high ability, two with moderate ability, and two with low ability. Students with high ability were those who scored ≥ 80, those with moderate ability scored between 70–80, and those with low ability scored < 70 on the initial mathematics test. The details of the selected subjects are presented in the following table.

Table 3. Research Subjects Based on Test 1 Result

No.	Initials of Subjects	Test 1 Result	Category
1	IZ, CT	> 80	High
2	KV, FT	70–80	Moderate
3	AB, FM	< 70	Low

Subsequently, the six selected subjects were administered a diagnostic test covering all components of number sense and mental arithmetic, followed by an interview session. The diagnostic test consisted of 10 items: 5 items measuring number sense, representing all of its components, and 5 additional items assessing mental arithmetic skills. During the administration of the number sense diagnostic test, students were asked to complete each item according to the instructions provided. For items related specifically to mental arithmetic skills, participants were instructed not to use any external calculating aids, such as pens or paper, ensuring that all computations were performed mentally.

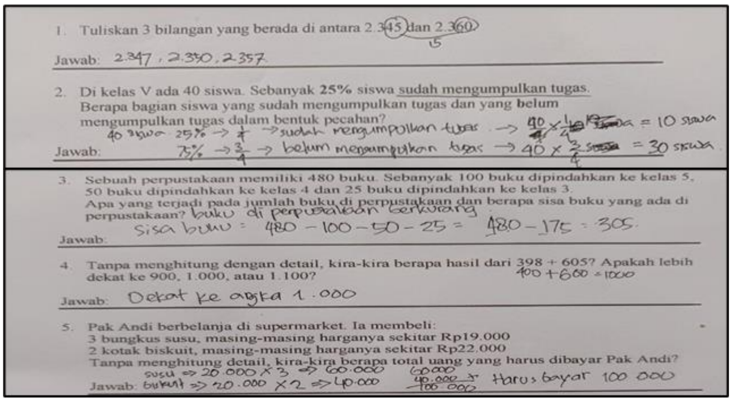
Number Sense in Terms of Mathematical Ability

The components of number sense may vary depending on the mathematics curriculum, grade level, and the content being taught (Birgin & Peker, 2024). In this study, the number sense components used to evaluate students' numerical sensitivity are based on the theoretical framework developed by McIntosh et al. (1992), McIntosh et al. (1997), and Singh (2009). Table 4 illustrates the achievement levels of each number sense component across different categories of students' mathematical ability. The data are presented systematically to provide a clear comparison of students' performance on each component.

Table 4. Achievement Levels for Number Sense Components

Number Sense	High Ability	Moderate Ability	Low Ability
Number Concepts	• Strong Mastery • Provide Correct Answer	• Generally Good • Different Result of each subject. One answered 3/3 correctly, another achieved 2/3	• Needs Improvement • Incomplete Understanding (2/3 items)
Multiple Representations	• Correct Responses • Understand Equivalence	• Conceptual Understanding • Misinterpreted Question	• Insufficient Understanding • Irrelevant Responses
Effects of Operations	• Accurate Understanding • Correct Answer	• Partial Understanding • Calculation Errors	• General Understanding • Inaccuracies in Calculation
Equivalent Representations	• Proficient Estimation • Proficient Rounding	• Proficient Estimation • Proficient Rounding	• Capable of Rounding • Unable to Estimate
Calculation Strategies	• Successful Mental Strategies • Accurate Estimation	• No Mental Strategies • Reliance on Written Work	• No Mental Strategies • Unable to Estimate

Based on the table, mathematical ability aligns closely with number sense mastery. While low and moderate-ability students exhibited limited to partial mastery with varying degrees of inaccuracy, high-ability students demonstrated a remarkably strong command across all components. This high proficiency is particularly evident in the documented work of subjects IZ and CT, who consistently applied well-developed number sense skills.


1. Tuliskan 3 bilangan yang berada di antara 2.345 dan 2.360

Jawab: 2.347, 2.350, 2.357

2. Di kelas V ada 40 siswa. Sebanyak **25%** siswa sudah mengumpulkan tugas. Berapa bagian siswa yang sudah mengumpulkan tugas dan yang belum mengumpulkan tugas dalam bentuk pecahan?

Jawab: 40 siswa 25% → 1/4 → sudah mengumpulkan tugas → 40 × 1/4 = 10 siswa
75% → 3/4 → belum mengumpulkan tugas → 40 × 3/4 = 30 siswa

3. Sebuah perpustakaan memiliki 480 buku. Sebanyak 100 buku dipindahkan ke kelas 5, 50 buku dipindahkan ke kelas 4 dan 25 buku dipindahkan ke kelas 3. Apa yang terjadi pada jumlah buku di perpustakaan dan berapa sisa buku yang ada di perpustakaan?

Jawab: buku di perpustakaan berkurang
sisa buku = 480 − 100 − 50 − 25 = 480 − 175 = 305.

4. Tanpa menghitung dengan detail, kira-kira berapa hasil dari 398 + 605? Apakah lebih dekat ke 900, 1.000, atau 1.100?

400 + 600 = 1000

Jawab: Dekat ke angka 1.000

5. Pak Andi berbelanja di supermarket. Ia membeli:
3 bungkus susu, masing-masing harganya sekitar Rp19.000
2 kotak biskuit, masing-masing harganya sekitar Rp22.000
Tanpa menghitung detail, kira-kira berapa total uang yang harus dibayar Pak Andi?

Jawab: susu ⇒ 20.000 × 3 ⇒ 60.000
biskuit ⇒ 20.000 × 2 ⇒ 40.000
60.000 + 40.000 = 100.000 Harus bayar 100.000

Figure 1. The work result of subject IZ and CT

Supporting evidence from the interviews further illustrates the practical application of these skills among high-ability subjects. For instance, subject IZ accurately identified a numerical interval of 15 between 2,345 and 2,360 (NS1/KMT-IZ) and demonstrated a clear understanding of equivalence by converting 25% to one-fourth to seamlessly deduce a remaining three-fourths (NS2/KMT-IZ). Similarly, subject CT exhibited advanced estimation strategies by rounding numbers to the nearest values. This approach was effectively used to simplify both addition-bringing an estimated sum closer to 1,000 (NS4/KMT-CT) and multiplication, allowing the subject to easily determine the approximate total cost of an item (NS5/KMT-CT).

These documented strategies confirm that individuals with strong number sense can flexibly apply their numerical knowledge to various mathematical problem-solving situations. The ability of these subjects to execute such efficient methods highlights the crucial role that cognitive capacity plays in complex mathematical thinking and problem-solving (Gallagher, 2019; McCoach & Flake, 2018). Furthermore, these findings strongly align with previous studies asserting that students with high mathematical ability naturally possess a more robust number sense, thereby granting them a significant operational advantage over their less proficient peers (Demirci et al., 2023; Wang et al., 2017; Yang & Chang, 2023).

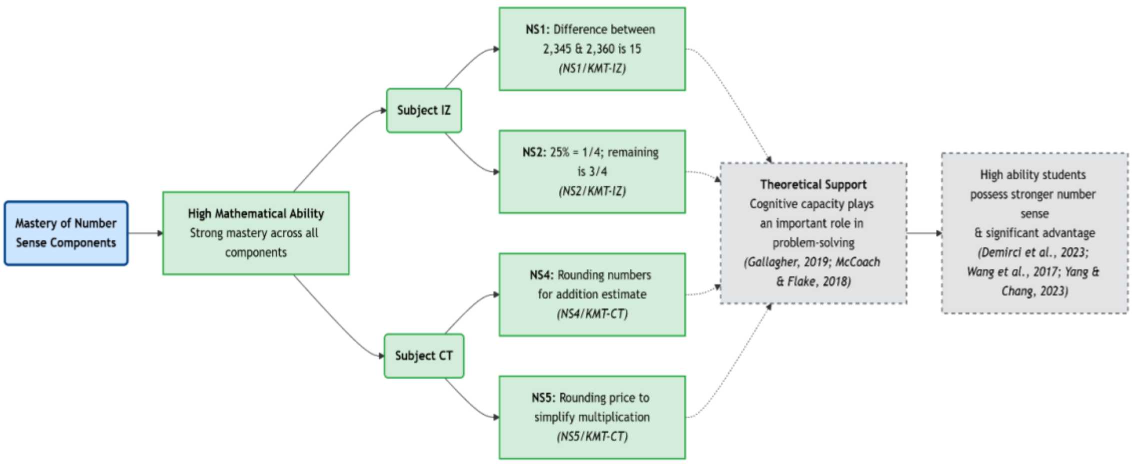


Figure 2. Mapping of Number Sense Strategies, Interview Evidence, and Theoretical Support for High Mathematical Ability Subjects.

Subjects with moderate mathematical ability demonstrated a reasonably good mastery of number sense skills when completing the tasks. Most components were fulfilled, although some were not maximized and did not lead to completely accurate responses. This is also reflected in the documented work of subjects KV and FT.

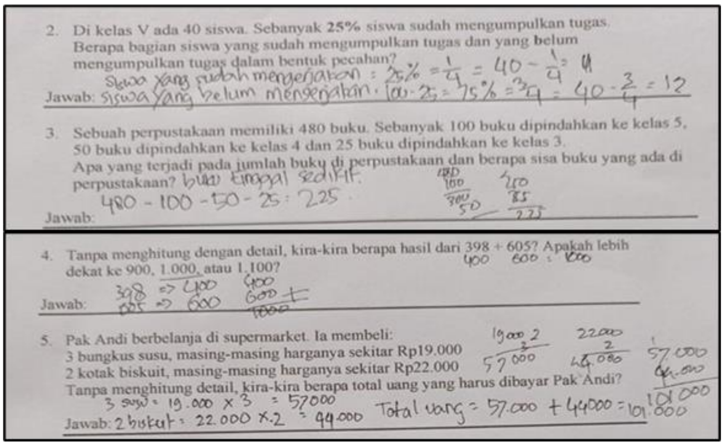


Figure 3. The work result of subject KV and FT

Students with moderate mathematical ability exhibited a reasonably good, albeit partial, mastery of number sense components. As reflected in the documented work and interviews of subjects KV and FT, these students successfully fulfilled several conceptual requirements but often struggled with procedural execution. For instance, subject KV correctly identified numerical equivalence—noting that 25% corresponds to one-fourth and the remaining 75% to three-fourths (NS2/KMS-KV)—and understood the conceptual effect of subtraction in a word problem, yet failed to perform the calculation accurately (NS3/KMS-KV). Similarly, while subject FT successfully applied rounding for estimation (NS4/KMS-FT), they struggled to utilize mental math in the subsequent task. Instead of applying the required mental calculation strategies, FT relied on detailed written calculations; although the final answer was correct, the procedure did not align with the intended problem-solving approach (NS5/KMS-FT).

These findings indicate that while moderate-ability students possess a foundational conceptual understanding, they often lack the procedural fluency and flexibility required to maximize their number sense. Their tendency to abandon mental strategies in favor of standard written algorithms-even when inefficient-is highly consistent with existing literature on number sense development (McIntosh et al., 1992; Reys et al., 1999). Furthermore, their susceptibility to calculation errors despite grasping the underlying concepts highlights a gap in operational adaptability (Markovits & Sowder, 1994). The specific cognitive profiles and strategic approaches of these moderate-ability subjects, along with the supporting theoretical frameworks, are visualized in Figure 4.

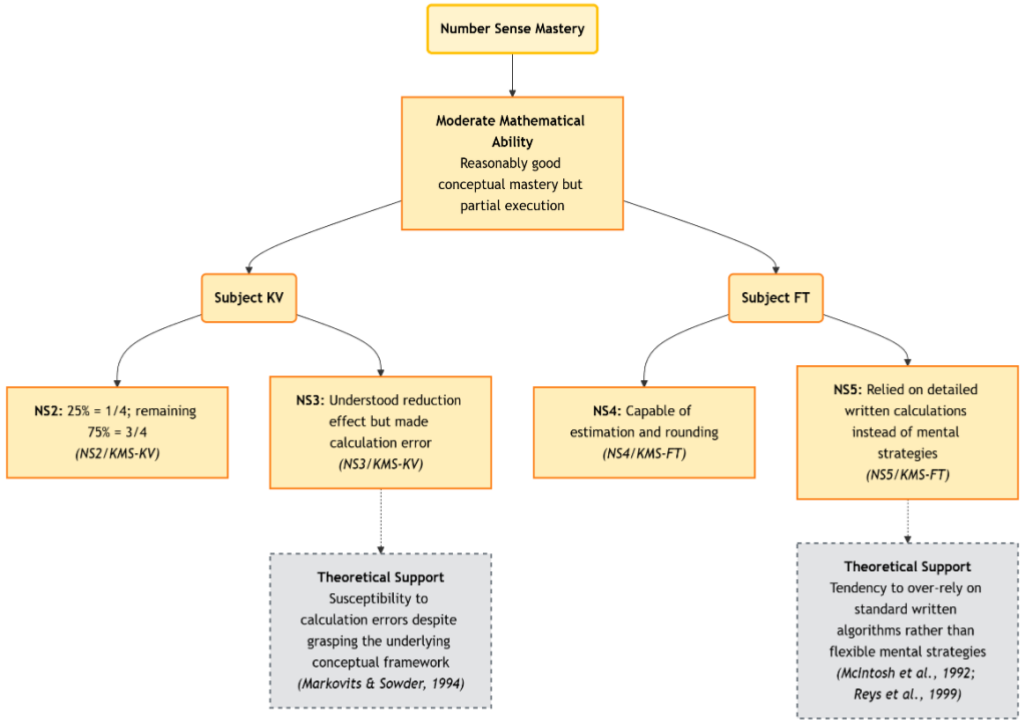


Figure 4. Mapping of Number Sense Strategies, Interview Evidence, and Theoretical Support for Moderate Mathematical Ability Subjects.

Subjects with low mathematical ability were unable to meet all components of number sense. Although some components could be partially understood, they consistently failed to produce correct answers. This condition was also reflected in the documented work of subjects AB and FM when completing the given tasks.

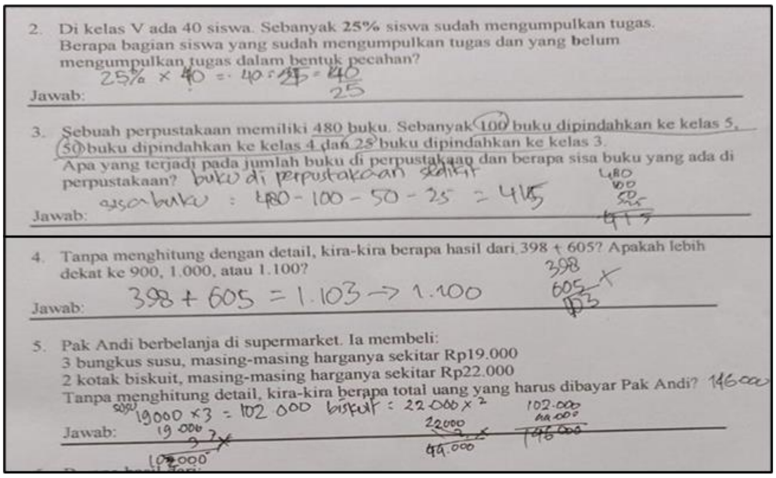


Figure 5. The work result of subject AB and FM

As evidenced by the documented work and interviews of subjects AB and FM, these students occasionally exhibited a partial grasp of mathematical contexts but consistently failed to produce accurate outcomes. For instance, subject FM explicitly stated an inability to recognize numerical equivalence between percentages and fractions (NS2/KMR-FM) and, despite understanding the conceptual effect of a subtraction operation, was unable to execute the calculation correctly (NS3/KMR-FM). Similarly, subject AB avoided utilizing mental strategies altogether, opting instead for detailed written calculations that ultimately yielded inaccurate results (NS5/KMR-AB). The specific cognitive obstacles and flawed strategic approaches of these low-ability subjects, supported by relevant theoretical frameworks, are visually mapped in Figure 6.

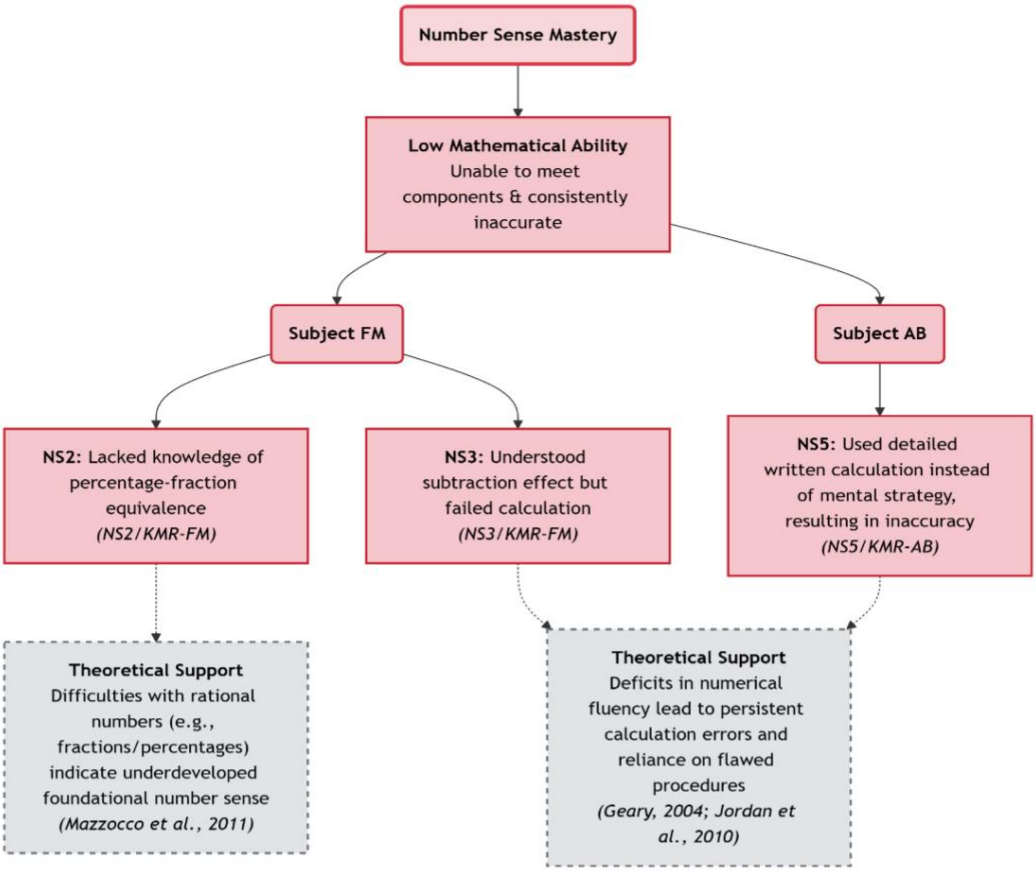


Figure 6. Mapping of Number Sense Strategies, Interview Evidence, and Theoretical Support for Low Mathematical Ability Subjects.

These findings highlight a profound deficit in foundational number sense, where an absence of conceptual fluency directly impairs procedural execution. The persistent inaccuracies and reliance on flawed calculation methods observed in these subjects strongly align with established research on low-achieving mathematics students (Geary, 2004; Jordan et al., 2010). Furthermore, their specific difficulties with rational numbers, such as percentage-fraction equivalence, and their inability to apply flexible mental strategies are hallmark indicators of underdeveloped number sense (Mazzocco et al., 2011).

In conclusion, the empirical evidence from both student task performance and interview data establishes a robust positive correlation between mathematical ability and number sense proficiency. Students with higher mathematical abilities consistently exhibit stronger, more flexible number sense, whereas those with lower abilities display significant deficits. These findings highlight that number sense is a foundational driver of mathematical success, equipping students with the cognitive flexibility required for advanced problem-solving and creative mathematical thinking (Boaler, 2015; Schneider et al., 2017). Consequently, to improve overall mathematical attainment, there is a critical need for educators to implement richer, targeted instructional strategies that explicitly cultivate number sense across all ability levels (Clements & Sarama, 2014).

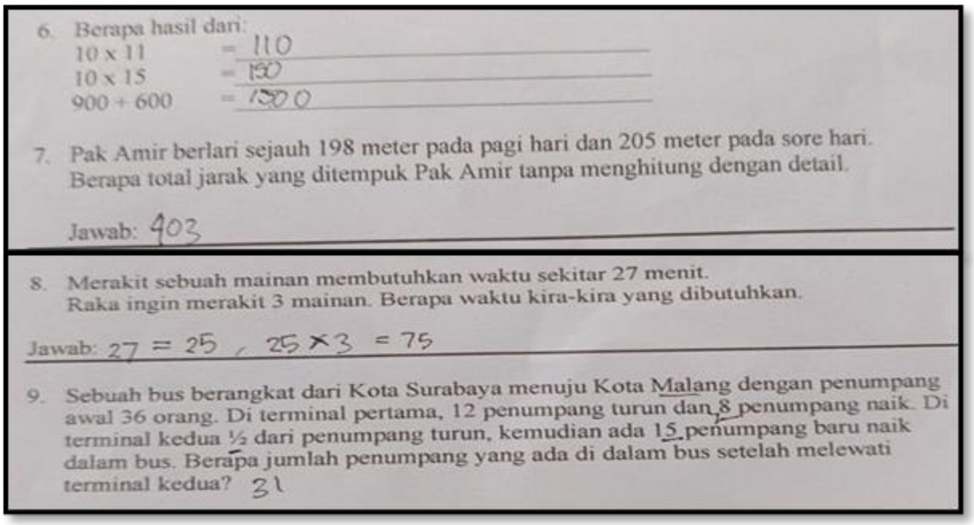
Mental Arithmetic in Terms of Mathematical Ability

Mental arithmetic is a calculation method performed by transferring the computational process into the mind, thereby eliminating dependence on external calculation tools such as calculators, computers, or written notes. In this context, the mental arithmetic component includes accuracy of results, the use of mental strategies, the ability to make estimations, the capacity to solve multi-step problems, and consistency throughout the calculation process. In solving the items that involved mental arithmetic, instructions were given to ensure that subjects did not use any calculating aids, including written scratch work. Thus, the mental arithmetic component could be observed through students' written work and the results of the interviews conducted. The following section presents a description of the achievement of each number sense component, analyzed based on the subjects' levels of mathematical ability. This explanation is presented systematically in the table below.

Table 5. Description of Achievement Levels for Mental Arithmetic Components

Component	High Ability	Moderate Ability	Low Ability
Result Accuracy	• Highly accurate • Solved all correctly without written scratch work.	• Partially accurate • Answered 2 out of 3 correctly without scratch work.	• Inaccurate • Answered only 1 out of 3 correctly without scratch work.
Mental Strategies	• Applied flexible strategies (e.g., rounding to nearest value and adjusting the difference) for 3-digit numbers.	• Attempted to use mental strategies, but the execution often led to inaccurate results.	• Did not use mental strategies • Relied on standard detailed calculations • Resulting in errors.
Estimation	• Produced appropriate estimations (e.g., rounding 27 to 25).	• Produced appropriate estimations (e.g., rounding 27 to 25).	• Produced inaccurate estimations (e.g., rounding 27 to 30).
Multi-step Problem Solving	• Successfully solved multi-step problems, despite displaying brief cognitive strain/temptation to use scratch work.	• Failed to solve; stopped midway due to the inability to use written scratch work.	• Failed to solve; stopped midway due to the inability to use written scratch work.
Consistency in Calculation	• Strong consistency in applying rounding values • Adjustment strategies mentally.	• Exhibited inconsistent strategy application • Attempted rounding but failed to maintain accuracy.	• Displayed consistent inability to use mental math • Persisting with detailed calculations that remained incorrect.

Based on the table, mental arithmetic proficiency aligns closely with overall mathematical ability. High-ability subjects, notably IZ and CT, demonstrated exceptional mastery by accurately performing calculations without any written aids. In contrast, moderate-ability students attempted mental strategies but frequently produced inaccurate results, while low-ability students lacked these strategies altogether, leading to consistently incorrect answers.


6. Berapa hasil dari:
10 x 11 = 110
10 x 15 = 150
900 + 600 = 1500

7. Pak Amir berlari sejauh 198 meter pada pagi hari dan 205 meter pada sore hari. Berapa total jarak yang ditempuk Pak Amir tanpa menghitung dengan detail.

Jawab: 403

8. Merakit sebuah mainan membutuhkan waktu sekitar 27 menit. Raka ingin merakit 3 mainan. Berapa waktu kira-kira yang dibutuhkan.

Jawab: 27 = 25 , 25 x 3 = 75

9. Sebuah bus berangkat dari Kota Surabaya menuju Kota Malang dengan penumpang awal 36 orang. Di terminal pertama, 12 penumpang turun dan 8 penumpang naik. Di terminal kedua ½ dari penumpang turun, kemudian ada 15 penumpang baru naik dalam bus. Berapa jumlah penumpang yang ada di dalam bus setelah melewati terminal kedua? 31

Figure 7. The work result of subject IZ and CT

This evidence is further supported by the interview results with the research subjects. Subject IZ stated that they simply added a zero to the number being multiplied by 10 (MA1/KMT-IZ). IZ also explained how they solved item number 7 by using a mental strategy, namely rounding both numbers to the nearest value and then adding or subtracting the remaining difference, which made the calculation easier (MA2/KMT-IZ). Subject CT mentioned experiencing some difficulty when working on item number 9 because they were not allowed to use written scratch work, and all calculations had to be done mentally. Nevertheless, subject CT was able to answer the question correctly and accurately (MA4/KMT-CT).

Students with high mathematical ability tend to be more flexible in performing calculations, especially when applying mental arithmetic strategies. This aligns with Feronika’s (2025) claim that students who regularly use this approach generally complete arithmetic operations more quickly compared to those who still rely on conventional calculation methods. Reys and Bestgen (1981) also stated that mental calculation strategies involve a cognitive process carried out rapidly without written tools, producing logical answers that closely approximate the actual results. These findings indicate that high mathematical ability supports more effective use of mental arithmetic strategies, thereby increasing efficiency and accuracy in the calculation process. These approaches followed by the theoretical will be visualized in Figure 8.

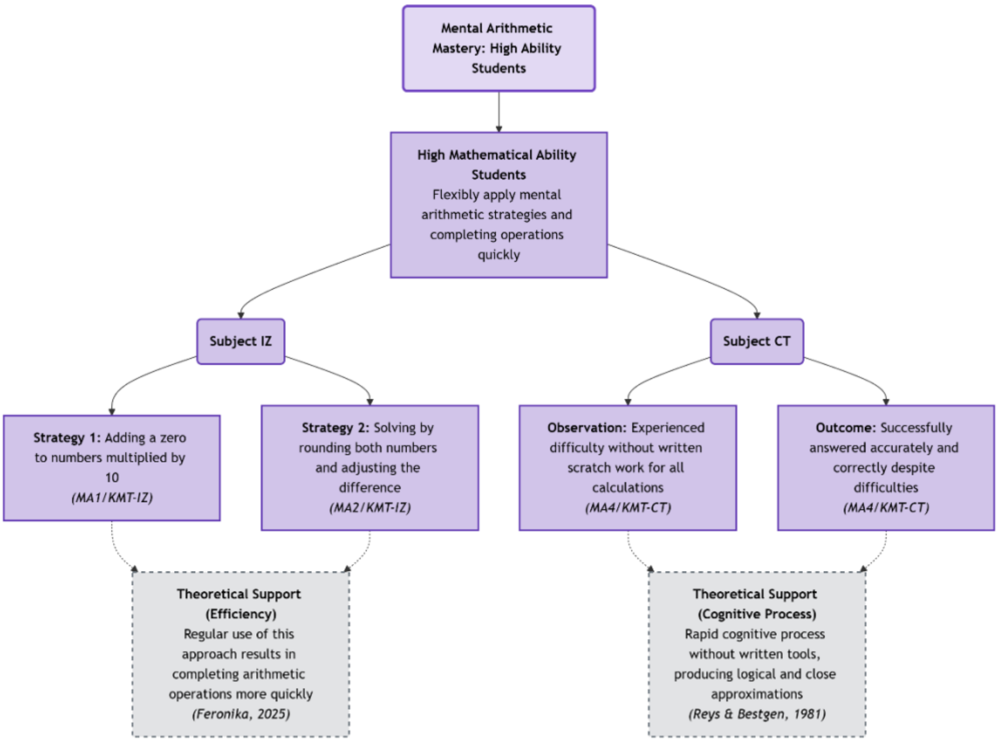
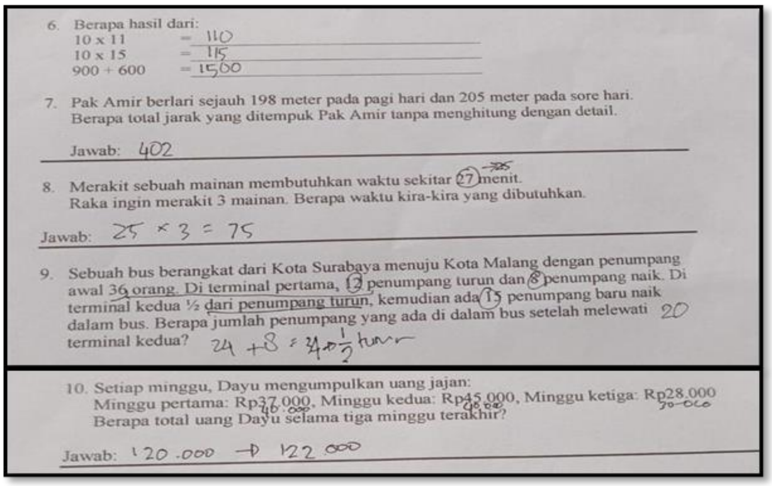


Figure 8. Mapping of Mental Arithmetic Strategies, Interview Evidence, and Theoretical Support for High Mathematical Ability Subjects

Subjects with moderate mathematical ability demonstrated attempts to perform almost all components of mental arithmetic skills, although several errors were still observed. This can be seen in the documentation study of the worksheet from subjects with moderate mathematical ability as follows.


6. Berapa hasil dari:
10 x 11 = 110
10 x 15 = 115
900 + 600 = 1500

7. Pak Amir berlari sejauh 198 meter pada pagi hari dan 205 meter pada sore hari. Berapa total jarak yang ditempuh Pak Amir tanpa menghitung dengan detail.

Jawab: 402

8. Merakit sebuah mainan membutuhkan waktu sekitar 27 menit. Raka ingin merakit 3 mainan. Berapa waktu kira-kira yang dibutuhkan.

Jawab: 25 × 3 = 75

9. Sebuah bus berangkat dari Kota Surabaya menuju Kota Malang dengan penumpang awal 36 orang. Di terminal pertama, 12 penumpang turun dan 8 penumpang naik. Di terminal kedua ½ dari penumpang turun, kemudian ada 15 penumpang baru naik dalam bus. Berapa jumlah penumpang yang ada di dalam bus setelah melewati terminal kedua?

10. Setiap minggu, Dayu mengumpulkan uang jajan:
Minggu pertama: Rp37.000, Minggu kedua: Rp45.000, Minggu ketiga: Rp28.000
Berapa total uang Dayu selama tiga minggu terakhir?

Jawab: 120.000 → 122.000

Figure 9. The work result of subject KV dan FT

As reflected in the documented worksheets and interview data, these students comprehended the necessity of mental strategies but struggled with execution. For instance, subject KV effectively applied an estimation strategy by rounding the number 27 to 25, which was then multiplied by 3 to estimate the required time for toy assembly (MA3/KMS-KV). Meanwhile, subject FT attempted to apply mental strategies for item number 10 but experienced significant difficulty due to the restriction against making written notes. Consequently, this cognitive strain led to an inaccurate final answer (MA5/KMS-FT).

These findings indicate that while moderate-ability students can select appropriate mental approaches, such as rounding for estimation (Sowder, 1992), their procedural execution is often hindered by cognitive overload. Subject FT's difficulty highlights the critical role of working memory capacity in mental arithmetic; without the ability to offload intermediate calculation steps onto paper, moderate students frequently lose track of numerical operations, resulting in errors (Heirdsfield & Cooper, 2004; Peng et al., 2016). The cognitive processes and theoretical connections explaining these moderate-ability students' mental arithmetic performance are visualized in Figure 10.

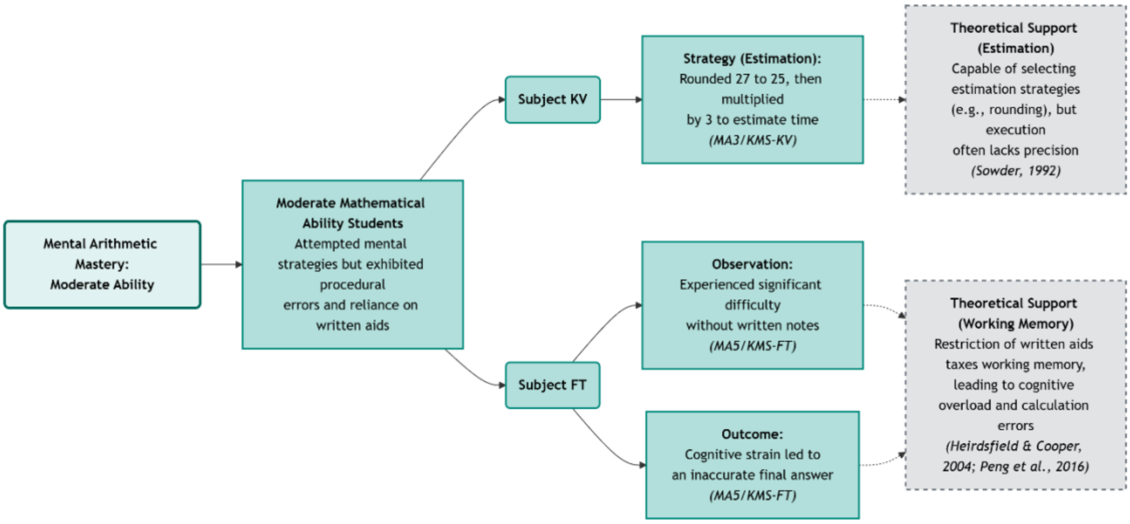
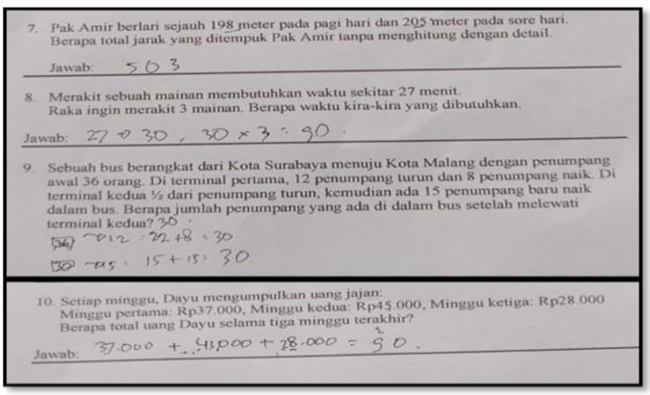


Figure 10. Mapping of Mental Arithmetic Strategies, Interview Evidence, and Theoretical Support for Moderate Mathematical Ability Subjects

Subjects with low mathematical ability experienced difficulties in meeting all components of mental arithmetic skills. This challenge was exacerbated by the instruction prohibiting the use of paper for scratch work, which made the calculation process more demanding. As a result, in addition to being unable to fulfill the mental arithmetic components, subjects with low mathematical ability frequently produced incorrect final answers. This was evidenced by their documented work on the items related to mental arithmetic components.


7. Pak Amir berlari sejauh 198 meter pada pagi hari dan 205 meter pada sore hari. Berapa total jarak yang ditempuk Pak Amir tanpa menghitung dengan detail.

Jawab: 503

8. Merakit sebuah mainan membutuhkan waktu sekitar 27 menit. Raka ingin merakit 3 mainan. Berapa waktu kira-kira yang dibutuhkan.

Jawab: 27 → 30 , 30 × 3 = 90 .

9. Sebuah bus berangkat dari Kota Surabaya menuju Kota Malang dengan penumpang awal 36 orang. Di terminal pertama, 12 penumpang turun dan 8 penumpang naik. Di terminal kedua ½ dari penumpang turun, kemudian ada 15 penumpang baru naik dalam bus. Berapa jumlah penumpang yang ada di dalam bus setelah melewati terminal kedua? 30

10. Setiap minggu, Dayu mengumpulkan uang jajan:
Minggu pertama: Rp37.000, Minggu kedua: Rp45.000, Minggu ketiga: Rp28.000
Berapa total uang Dayu selama tiga minggu terakhir?

Jawab: 37.000 + 45.000 + 28.000 = 90 .

Figure 11. The work result of subject AB dan FM

Interview data from subject AB vividly illustrates these cognitive deficits. When attempting estimation, AB applied a flawed strategy by inaccurately rounding 27 to 30 (MA3/KMR-AB). Furthermore, AB explicitly reported severe difficulties when solving item number 9, noting that the inability to perform written calculations forced them to mentally retain multiple computation steps-a task they could not sustain (MA4/KMR-AB).

These findings highlight a critical breakdown in both foundational number sense and executive functioning among low-ability students. The inaccurate estimation (rounding 27 to 30) reflects a poorly developed understanding of numerical magnitude (Siegler & Lortie-Forgues, 2014). Moreover, subject AB's inability to mentally hold and manipulate numbers without written aids points to a severe limitation in working memory capacity. Extensive research confirms that deficits in working memory prevent students from retaining intermediate computational steps, leading directly to calculation failures in mental arithmetic (Cragg et al., 2017; Passolunghi & Costa, 2019). The cognitive obstacles and corresponding theoretical explanations for low-ability subjects are visually mapped in Figure 12.

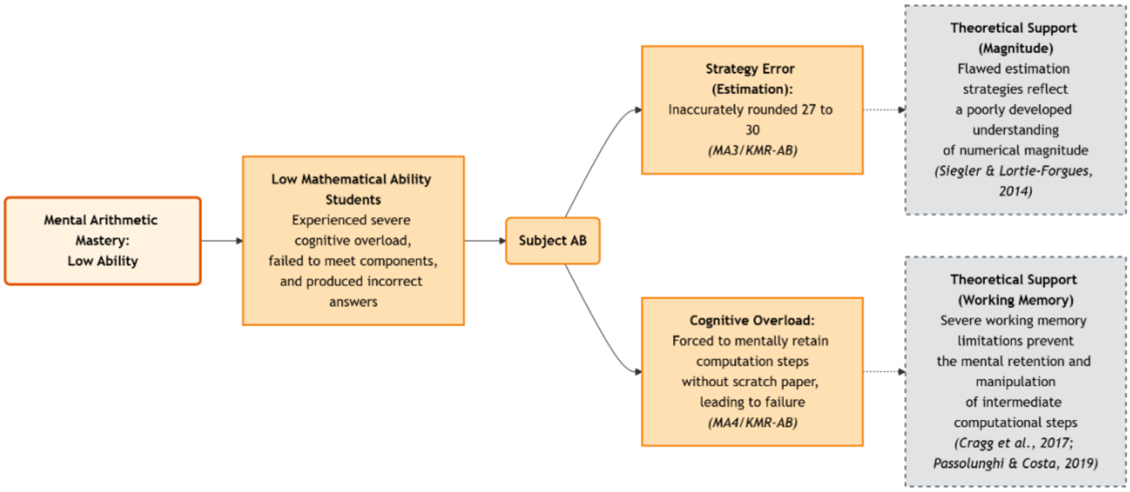


Figure 12. Mapping of Mental Arithmetic Strategies, Interview Evidence, and Theoretical Support for Moderate Mathematical Ability Subjects

Based on the analysis of the documentation and interview data, a clear distinction can be observed among subjects with high, moderate, and low mathematical ability in their calculation processes, particularly in terms of mental arithmetic skills. High-ability subjects were able to complete all items requiring mental arithmetic strategies very effectively. Moderate-ability subjects demonstrated performance that nearly fulfilled all components, although several minor errors were still identified. Conversely, low-ability subjects encountered significant difficulties in applying mental arithmetic strategies to complete the calculations.

These differences indicate that mastery of mental arithmetic plays an important role in improving the quality of students' computational processes. In line with this, Dianto (2018) explains that the development of mental arithmetic offers various benefits, including: (1) optimizing brain function, (2) enhancing imagination, creativity, logic, systematic thinking patterns, concentration, and memory, (3) increasing speed, accuracy, and precision in thinking, (4) strengthening sensitivity to spatial relationships as a result of visualizing the abacus mentally, (5) supporting students who frequently face difficulties memorizing multiplication facts, and (6) fostering self-confidence and a positive mental attitude, particularly when dealing with mathematical tasks or problems.

Overall, the results of this study emphasize that mental arithmetic skills not only contribute to improving fluency in computation but also support the strengthening of broader cognitive abilities. The performance differences among subjects with high, moderate, and low mathematical ability demonstrate that mastering mental arithmetic strategies is an essential skill that should be cultivated from an early age. With its numerous benefits—ranging from cognitive development and improved accuracy to the formation of a positive attitude toward mathematics—mental arithmetic represents a relevant approach to be integrated into instruction. These findings are expected to serve as a basis for the development of more effective instructional strategies in the future.

CONCLUSION

Conclusion

From the findings presented in this study, it is evident that number sense and mental arithmetic abilities play a significant role in facilitating students' performance when solving mathematical problems. Based on the results, it can be concluded that subjects with high mathematical ability were able to meet all components of number sense and mental arithmetic effectively. Subjects with moderate mathematical ability fulfilled nearly all components, although several errors were still observed. Meanwhile, subjects with low mathematical ability experienced considerable difficulty in meeting all components of number sense and mental arithmetic. Thus, it can be inferred that the stronger a student's number sense and mental arithmetic skills, the better their mathematical performance tends to be.

Recommendations

This study focuses on the analysis of number sense and mental arithmetic in relation to mathematical ability among fifth-grade primary school students. For future research, it is recommended that this topic be further developed using different research models, such as experimental designs, mixed-method approaches, or longitudinal studies, to gain a deeper and more comprehensive understanding of the issues explored. In addition, future studies may involve students from different educational levels or examine the relationship between number sense and mental arithmetic with other variables, such as teaching strategies, cognitive styles, or the use of instructional media. These developments are expected to enrich the findings and contribute more broadly to the improvement of mathematical learning in primary education.

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